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A Hardy's Uncertainty Principle Lemma in Weak Commutation Relations of Heisenberg-Lie Algebra

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Abstract. In this article we consider linear operators satisfying a generalized commutation relation of a type of the Heisenberg-Lie algebra. It is proven that a generalized inequality of the Hardy's uncertainty principle lemma follows. Its applications to time operators and abstract Dirac operators are also investigated.

Key words : weak commutation relations, Heisenberg-Lie algebra, time operators, Hamiltonians, time-energy uncertainty relation, Dirac operators, essential self-adjointness.

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1 Introduction and Results

In this article we investigate a norm-inequality of the linear operators which obey a generalized weak commutation relation of a type of the Heisenberg-Lie algebra, and consider its application to the theory of the time operator [7, 2], and an abstract Dirac operator. Let $\mathbf{X} = \{X_j\}_{j=1}^N$, $\mathbf{Y} = \{Y_j\}_{j=1}^N$ and $\mathbf{Z} = \{Z_j\}_{j=1}^N$ be symmetric operators on a Hilbert space \mathcal{H} . The weak commutator of operators A and B is defined for $\psi \in \mathcal{D}(A) \cap \mathcal{D}(B)$ and $\phi \in \mathcal{D}(A^*) \cap \mathcal{D}(B^*)$ by

$$[A, B]^w(\phi, \psi) = (A^* \phi, B\psi) - (B^* \phi, A\psi).$$

Here the inner product has a linearity of $(\eta, \alpha\psi + \beta\phi) = \alpha(\eta, \psi) + \beta(\eta, \phi)$ for $\alpha, \beta \in \mathbf{C}$. We assume that $(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$ satisfies the following conditions.

(A.1) Z_j , $1 \leq j \leq N$, is bounded operator.

(A.2) Let $\mathcal{D}_{\mathbf{X}} = \cap_{j=1}^N \mathcal{D}(X_j)$ and $\mathcal{D}_{\mathbf{Y}} = \cap_{j=1}^N \mathcal{D}(Y_j)$. It follows that for $\phi, \psi \in \mathcal{D}_{\mathbf{X}} \cap \mathcal{D}_{\mathbf{Y}}$,

$$\begin{aligned} [X_j, Y_l]^w(\phi, \psi) &= \delta_{j,l}(\phi, iZ_j\psi), \\ [X_j, Z_l]^w(\phi, \psi) &= [Y_j, Z_l]^w(\phi, \psi) = 0 \\ [X_j, X_l]^w(\phi, \psi) &= [Y_j, Y_l]^w(\phi, \psi) = [Z_j, Z_l]^w(\phi, \psi) = 0. \end{aligned}$$

Note that $[Z_j, Z_l]\psi = 0$ follows for $\psi \in \mathcal{H}$, since Z_j , $j = 1, \dots, N$, is bounded. In this article we consider an generalization of the inequality

$$\int_{\mathbf{R}^N} \frac{1}{|\mathbf{r}|^2} |u(\mathbf{r})|^2 d\mathbf{r} \leq \frac{4}{(N-2)^2} \int_{\mathbf{R}^N} |\nabla u(\mathbf{r})|^2 d\mathbf{r}, \quad N \geq 3.$$

This inequality is a basic one of Hardy's uncertainty principle inequalities. For Hardy's uncertainty inequalities, refer to e.g. [5, 6, 13].

Let us introduce the additional conditions.

(A.3) X_j is self-adjoint for all $1 \leq j \leq N$.

(A.4) X_i and Z_l strongly commutes for all $1 \leq j \leq N$ and $1 \leq l \leq N$.

Since Z_j , $j = 1, \dots, N$, is bounded self-adjoint operator, we can set $\lambda_{\min}(\mathbf{Z})$ and $\lambda_{\max}(\mathbf{Z})$ by

$$\begin{aligned}\lambda_{\min}(\mathbf{Z}) &= \min_{1 \leq j \leq N} \inf \sigma(Z_j), \\ \lambda_{\max}(\mathbf{Z}) &= \max_{1 \leq j \leq N} \sup \sigma(Z_j),\end{aligned}$$

where $\sigma(O)$ denotes the spectrum of the operator O .

Theorem 1 *Assume (A.1)-(A.4). Let $\Psi \in \mathcal{D}(|\mathbf{X}|^{-1}) \cap \mathcal{D}_{\mathbf{X}} \cap \mathcal{D}_{\mathbf{Y}}$. Then the following (1) and (2) hold*
(1) *If $N\lambda_{\min}(\mathbf{Z}) - 2\lambda_{\max}(\mathbf{Z}) > 0$, it follows that*

$$\left\| |\mathbf{X}|^{-1} \Psi \right\|^2 \leq \frac{4}{(N\lambda_{\min}(\mathbf{Z}) - 2\lambda_{\max}(\mathbf{Z}))^2} \sum_{j=1}^N \left\| Y_j \Psi \right\|^2. \quad (1)$$

(2) *If $2\lambda_{\min}(\mathbf{Z}) - N\lambda_{\max}(\mathbf{Z}) > 0$, it follows that*

$$\left\| |\mathbf{X}|^{-1} \Psi \right\|^2 \leq \frac{4}{(2\lambda_{\min}(\mathbf{Z}) - N\lambda_{\max}(\mathbf{Z}))^2} \sum_{j=1}^N \left\| Y_j \Psi \right\|^2. \quad (2)$$

Before proving Theorem 1, let us consider the replacement of \mathbf{X} and \mathbf{Y} in Theorem 1. Let us introduce the following conditions substitute for **(A.3)** and **(A.4)**.

(A.5) Y_j is self-adjoint for all $1 \leq j \leq N$.

(A.6) Y_i and Z_l strongly commutes for all $1 \leq j \leq N$ and $1 \leq l \leq N$.

It is seen from **(A.2)**, that

$$[Y_j, X_l]^w(\phi, \psi) = \delta_{j,l}(\phi, i(-Z_j)\psi), \quad \phi, \psi \in \mathcal{D}_{\mathbf{X}} \cap \mathcal{D}_{\mathbf{Y}}. \quad (3)$$

Note that $\inf \sigma(-Z_j) = -\sup \sigma(Z_j)$ and $\sup \sigma(-Z_j) = -\inf \sigma(Z_j)$ follow. Then we obtain a following corollary :

Corollary 2 Assume (A.1)-(A.2) and (A.5)-(A.6). Let $\Psi \in \mathcal{D}(|\mathbf{Y}|^{-1}) \cap \mathcal{D}_{\mathbf{X}} \cap \mathcal{D}_{\mathbf{Y}}$. Then the following (1) and (2) hold.

(1) If $2\lambda_{\min}(\mathbf{Z}) - N\lambda_{\max}(\mathbf{Z}) > 0$, it follows that

$$\left\| |\mathbf{Y}|^{-1}\Psi \right\| \leq \frac{4}{(2\lambda_{\min}(\mathbf{Z}) - N\lambda_{\max}(\mathbf{Z}))^2} \sum_{j=1}^N \left\| X_j \Psi \right\|^2. \quad (4)$$

(2) If $N\lambda_{\min}(\mathbf{Z}) - 2\lambda_{\max}(\mathbf{Z}) > 0$, it follows that

$$\left\| |\mathbf{Y}|^{-1}\Psi \right\| \leq \frac{4}{(N\lambda_{\min}(\mathbf{Z}) - 2\lambda_{\max}(\mathbf{Z}))^2} \sum_{j=1}^N \left\| X_j \Psi \right\|^2. \quad (5)$$

(Proof of Theorem 1)

(1) Let $\Psi \in \mathcal{D}(|\mathbf{X}|^{-1}) \cap \mathcal{D}_{\mathbf{X}} \cap \mathcal{D}_{\mathbf{Y}}$. For $\varepsilon > 0$ and $t > 0$, it is seen that

$$\left\| (Y_j - itX_j(\mathbf{X}^2 + \varepsilon)^{-1}) \Psi \right\|^2 = \|Y_j \Psi\|^2 - it[Y_j, X_j(\mathbf{X}^2 + \varepsilon)^{-1}]^w(\Psi, \Psi) + t^2 \|X_j(\mathbf{X}^2 + \varepsilon)^{-1} \Psi\|^2. \quad (6)$$

We see that

$$[Y_j, X_j(\mathbf{X}^2 + \varepsilon)^{-1}]^w(\Psi, \Psi) = [Y_j, X_j]^w(\Psi, (\mathbf{X}^2 + \varepsilon)^{-1}\Psi) + [Y_j, (\mathbf{X}^2 + \varepsilon)^{-1}]^w(X_j \Psi, \Psi). \quad (7)$$

From (A.2) and (A.4), we obtain that

$$[Y_j, X_j]^w(\Psi, (\mathbf{X}^2 + \varepsilon)^{-1}\Psi) = -i((\mathbf{X}^2 + \varepsilon)^{-1/2}\Psi, Z_j(\mathbf{X}^2 + \varepsilon)^{-1/2}\Psi). \quad (8)$$

Note that for a symmetric operator A and the non-negative symmetric operator B , the resolvent formula $[A, (B + \lambda)^{-1}]^w(v, u) = [B, A]^w((B + \lambda)^{-1}v, (B + \lambda)^{-1}u)$ for $\lambda > 0$ follows. Then by using this formula, (A.2) and (A.4) yield that

$$[Y_j, (\mathbf{X}^2 + \varepsilon)^{-1}]^w(X_j \Psi, \Psi) = 2i(X_j(\mathbf{X}^2 + \varepsilon)^{-1}u, Z_j X_j(\mathbf{X}^2 + \varepsilon)^{-1}u) \quad (9)$$

Since $\left\| (Y_j - itX_j(\mathbf{X}^2 + \varepsilon)^{-1}) u \right\|^2 \geq 0$ and $t > 0$, we see from (7), (8) and (9) that

$$\begin{aligned} & \|Y_j \Psi\|^2 \\ & \geq -t^2 \|X_j(\mathbf{X}^2 + \varepsilon)^{-1}u\|^2 + t((\mathbf{X}^2 + \varepsilon)^{-1/2}\Psi, Z_j(\mathbf{X}^2 + \varepsilon)^{-1/2}u) - 2t(X_j(\mathbf{X}^2 + \varepsilon)^{-1}u, Z_j X_j(\mathbf{X}^2 + \varepsilon)^{-1}\Psi) \\ & \geq \left(-t^2 - 2t\lambda_{\max}(\mathbf{Z}) \right) \|X_j(\mathbf{X}^2 + \varepsilon)^{-1}u\|^2 + t\lambda_{\min}(\mathbf{Z}) \|(\mathbf{X}^2 + \varepsilon)^{-1/2}\Psi\|. \end{aligned} \quad (10)$$

Then we have that

$$\sum_{j=1}^N \|Y_j \Psi\|^2 \geq \left(-t^2 - 2t\lambda_{\max}(\mathbf{Z}) \right) \left\| |\mathbf{X}|(\mathbf{X}^2 + \varepsilon)^{-1}\Psi \right\|^2 + tN\lambda_{\min}(\mathbf{Z}) \|(\mathbf{X}^2 + \varepsilon)^{-1/2}\Psi\|. \quad (11)$$

Note that $\lim_{\varepsilon \rightarrow 0} \left\| |\mathbf{X}|(\mathbf{X}^2 + \varepsilon)^{-1}\Psi \right\|^2 = \left\| |\mathbf{X}|^{-1}\Psi \right\|^2$ and $\lim_{\varepsilon \rightarrow 0} \|(\mathbf{X}^2 + \varepsilon)^{-1/2}\Psi\| = \left\| |\mathbf{X}|^{-1}\Psi \right\| = 0$ follow from the spectral decomposition theorem. Then we have

$$\sum_{j=1}^N \|Y_j \Psi\|^2 \geq (-t^2 + (N\lambda_{\min}(\mathbf{Z}) - 2\lambda_{\max}(\mathbf{Z}))t) \left\| |\mathbf{X}|^{-1}\Psi \right\|^2. \quad (12)$$

By taking $t = \frac{N\lambda_{\min}(\mathbf{Z}) - 2\lambda_{\max}(\mathbf{Z})}{2} > 0$ in the right side of (12), we obtain **(1)**.

(2) By computing $\| (Y_j + itX_j(\mathbf{X}^2 + \varepsilon)^{-1}) \Psi \|^2$ for $t > 0$ and $\varepsilon > 0$, in a similar way of **(1)**, we see that

$$\begin{aligned} & \|Y_j \Psi\|^2 \\ & \geq -t^2 \|X_j(\mathbf{X}^2 + \varepsilon)^{-1}u\|^2 - t((\mathbf{X}^2 + \varepsilon)^{-1/2}\Psi, Z_j(\mathbf{X}^2 + \varepsilon)^{-1/2}u) + 2t(X_j(\mathbf{X}^2 + \varepsilon)^{-1}u, Z_jX_j(\mathbf{X}^2 + \varepsilon)^{-1}\Psi) \\ & \geq \left(-t^2 + 2t\lambda_{\min}(\mathbf{Z}) \right) \|X_j(\mathbf{X}^2 + \varepsilon)^{-1}u\|^2 - t\lambda_{\max}(\mathbf{Z}) \|(\mathbf{X}^2 + \varepsilon)^{-1/2}\Psi\|. \end{aligned} \quad (13)$$

Then by taking $\varepsilon \rightarrow 0$ in the right side of (13), it follows that

$$\sum_{j=1}^N \|Y_j \Psi\|^2 \geq \left(-t^2 + (2\lambda_{\min}(\mathbf{Z}) - N\lambda_{\max}(\mathbf{Z}))t \right) \| |\mathbf{X}|^{-1}\Psi \|. \quad (14)$$

By taking $t = \frac{(2\lambda_{\min}(\mathbf{Z}) - N\lambda_{\max}(\mathbf{Z}))}{2} > 0$ in (14), we obtain **(2)**. ■

2 Applications

2.1 Time-Energy Uncertainty inequality

In this subsection we consider an application to the theory of time operators [2, 7]. Let H , T , and C be linear operators on a Hilbert space H . It is said that H has the weak time operator T with the uncommutative factor C if (H, T, C) satisfy the following conditions.

(T.1) H and T are symmetric.

(T.2) C is bounded and self-adjoint.

(T.3) It follows that for $\phi, \psi \in \mathcal{D}(H) \cap \mathcal{D}(T)$,

$$[T, H]^w(\phi, \psi) = (\phi, C\psi).$$

(T.4)

$$\delta_C := \inf_{\psi \in (\ker C)^\perp \setminus \{0\}} \frac{|(\Psi, C\Psi)|}{\|\Psi\|^2} > 0.$$

Assume that (H, T, C) satisfies **(T.1)**-**(T.4)**. Then by using $\|Au\| \|Bu\| \geq |\operatorname{Im}(Au, Bu)| \geq \frac{1}{2} |[A, B]^w(u, u)|$, it is seen that (H, T, C) satisfies the time-energy uncertainty inequality ([2], Proposition 4.1):

$$\frac{\| (H - \langle H \rangle_\psi) \psi \| \| (T - \langle T \rangle_\psi) \psi \|}{\|\psi\|^2} \geq \frac{\delta_C}{2}, \quad \psi \in \mathcal{D}(H) \cap \mathcal{D}(T), \quad (15)$$

where $\langle O \rangle_\psi = (\psi, O\psi)$. From **(2)** in Theorem 1 and **(1)** in Corollary 2, we obtain another type of the inequality between T and H :

Corollary 3 (Time-Energy Uncertainty Inequalities)

Assume (T.1)-(T.3). Then the following (i) and (ii) hold.

(i) If T is self-adjoint, C and T strongly commute, and $\sup \sigma(C) < 2 \inf \sigma(C)$, it follows that for $\psi \in \mathcal{D}(|T|^{-1}) \cap \mathcal{D}(T) \cap \mathcal{D}(H)$,

$$\left\| |T|^{-1} \psi \right\| \leq \frac{2}{2 \inf \sigma(C) - \sup \sigma(C)} \left\| H \Psi \right\|. \quad (16)$$

(ii) If H is self-adjoint, C and H strongly commute, and $\sup \sigma(C) < 2 \inf \sigma(C)$, it follows that for $\psi \in \mathcal{D}(|H|^{-1}) \cap \mathcal{D}(H) \cap \mathcal{D}(T)$,

$$\left\| |H|^{-1} \psi \right\| \leq \frac{2}{2 \inf \sigma(C) - \sup \sigma(C)} \left\| T \Psi \right\|. \quad (17)$$

2.2 Abstract Dirac Operators with Coulomb Potential

Next let us consider the application to abstract Dirac operators. We consider the self-adjoint operators $\mathbf{P} = \{P_j\}_{j=1}^N$ and $\mathbf{Q} = \{Q_j\}_{j=1}^N$ on a Hilbert space \mathcal{H} . Let us set a subspace $\mathcal{D} \subset \cap_{j,l} (\mathcal{D}(P_j) \cap \mathcal{D}(Q_l))$. It is said that $(\mathcal{H}, \mathcal{D}, \mathbf{P}, \mathbf{Q})_N$ is the weak representation of the CCR with degree N , if \mathcal{D} is dense in \mathcal{H} and it follows that for $\phi, \psi \in \mathcal{D}$,

$$\begin{aligned} [P_j, Q_l]^w(\phi, \psi) &= i\delta_{j,l}(\phi, \psi), \\ [P_j, P_l]^w(\phi, \psi) &= [Q_j, Q_l]^w(\phi, \psi) = 0. \end{aligned}$$

Let us define an abstract Dirac operator as follows. Let $(\mathcal{H}, \mathcal{D}, \mathbf{P}, \mathbf{Q})_3$ be the weak representation of the CCR with degree three. Let $\mathbf{A} = \{A_j\}_{j=1}^3$ and B be the bounded self-adjoint operators on a Hilbert space \mathcal{K} . Here $\mathbf{A} = \{A_j\}_{j=1}^3$ and B satisfy the canonical anti-commutation relations $\{A_j, A_l\} = 2\delta_{j,l}$, $\{A_j, B\} = 0$, $B^2 = I_{\mathcal{K}}$ where $I_{\mathcal{K}}$ is the identity operator on \mathcal{K} . The state Hilbert space space is defined by $\mathcal{H}_{\text{Dirac}} = \mathcal{K} \otimes \mathcal{H}$. The free abstract Dirac operator is defined by

$$H_0 = \sum_{j=1}^3 A_j \otimes P_j + B \otimes M.$$

Here we assume the following condition.

(D.1) P_j and P_l strongly commute for $1 \leq j \leq 3$, $1 \leq l \leq 3$. P_j , $1 \leq j \leq 3$, and M strongly commute.

Then it is seen that $H_0^2 \Psi = (\mathbf{P}^2 + M^2) \Psi$ for $\Psi \in \mathcal{D}$. The abstract Dirac Operator with the Coulomb potential is defined by

$$H(\kappa) = H_0 + \kappa I_{\mathcal{K}} \otimes |\mathbf{Q}|^{-1},$$

where $\kappa \in \mathbf{R}$ is a parameter called the coupling constant. We assume that the following condition

(D.2) It follows that $\mathcal{D} \subset \mathcal{D}(|\mathbf{Q}|^{-1})$.

Then it follows from (1) in Theorem 1 that for $\psi \in \mathcal{D}$,

$$\|I_{\mathcal{K}} \otimes |\mathbf{Q}|^{-1} \psi\|^2 \leq 4 \sum_{j=1}^3 \|P_j \Psi\|^2 \leq 4 \|H_0 \Psi\|^2.$$

Hence by the Kato-Rellich theorem, we obtain the following corollary.

Corollary 4 Assume (D.1) and (D.2). Then for $|\kappa| < \frac{1}{2}$, $H(\kappa)$ is essentially self-adjoint on \mathcal{D} .

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